Dario Trinchero

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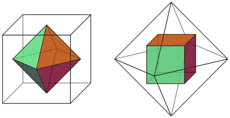
The Cave Wall:

Is a Necessarily Necessary God Possible?

The modal ontological argument

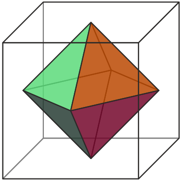
Stellenbosch University

August 2024



Talk Outline

1. Ontological argument



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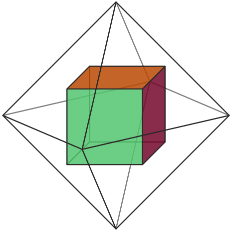

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Anselm's ontological argument

Definition (God)

God $:=$ "something than which nothing greater can be conceived"

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2. Such a being $>$ one whose existence is CONTINGENT

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Proslogion II

1. God can be CONCEIVED
2. That which can be conceived exists "in the intellect"
3. Existence "in reality" > existence in the intellect
4. ∴ God exists in reality

Proslogion III

1. A being whose existence is NECESSARY is CONCEIVABLE
2. Such a being > one whose existence is CONTINGENT
3. ∴ God NECESSARILY exists

Rebuttals

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$\longrightarrow$ Anselm's definition of God may be UNSATISFIABLE!

eg. $n :=$ largest integer
only says that IF n exists, it is largest

What remains?

Weighing Anselm vs. Kant, perhaps we only grant:

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Weighing Anselm vs. Kant, perhaps we only grant:

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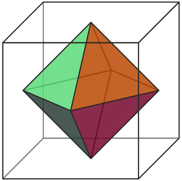
But this actually LOGICALLY IMPLIES that God exists !!

2. Propositional logic



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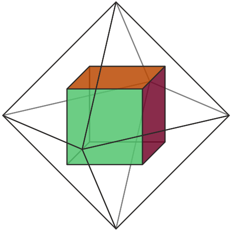

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FORMULAS are built using the alphabet:

- $P, Q, R, \ldots$ — propositional variables
- $\wedge, \vee, \neg\ \rightarrow$ — connectives
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Example (Well-formed formula)

$(P \rightarrow ((\neg Q) \wedge R)) \vee (\neg Q)$

Semantics are formalized in TRUTH TABLES.

Example (Truth table)

P	Q	$\neg P$	$\neg Q$	$P \wedge Q$	$\neg(P \wedge Q)$	$(\neg P) \vee (\neg Q)$
T	T					
T	F					
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Theorem ($\neg$, $\to$ are all you need)

- $P \vee Q \equiv (\neg P) \to Q$
- $P \wedge Q \equiv \neg(P \to (\neg Q))$

Axiom schemas

A $P \to (Q \to P)$

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Sequence of statements, each of which is:

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Definition (Proof)

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- the result of applying *modus ponens* on prior lines.

Syntax

Example proof

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$\vdash (P \to P)$ "$P \to P$ is a theorem / a tautology / provable from no assumptions"

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3. $(P \to (P \to P)) \to (P \to P)$ — by *modus ponens* 1 & 2
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4 $P \to (P \to P)$ by A $Q = P$

5 $P \to P$ by *modus ponens* 3 & 4

∎

Important meta-logic results

Deduction theorem, completeness & soundness

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$P \vdash Q$ *if and only if* $\vdash P \rightarrow Q$

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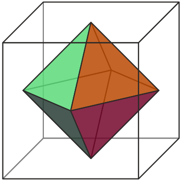
In particular, we assume we can prove any TAUTOLOGY.

3. Modal logic



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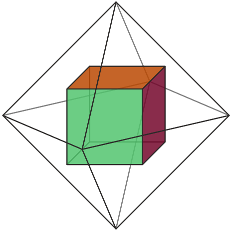

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Definition (Formulas)

MODAL LOGIC adds symbols:

- $\Box, \Diamond$ — MODAL OPERATORS

with interpretations:

- $\Box P$ — necessarily P — "P true in *every possible world*"
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Example (Modal statements)

- $\Box(1 + 1 = 2)$ **true**

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Example (Modal statements)

- $\Box(1+1=2)$ **true**
- $\Box$(flamingos are pink) **false**

Definition (Formulas)

Modal logic adds symbols:

- $\Box, \Diamond$ — modal operators

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Example (Modal statements)

- $\Box(1 + 1 = 2)$ — **true**
- $\Box$(flamingos are pink) — **false**
- $\Diamond$(Saul Kripke was president) — **true**

Deduction rule (rule of *necessitation*)

$$\frac{\vdash P}{\vdash \Box P}$$

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We could stop here $\longrightarrow$ MODAL LOGIC K

Syntax

. . . & the modal logic zoo

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For ALETHIC MODALITY, we conventionally also take:

T $\Box P \to P$ modal logic KT

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- **4** $\Box P \to \Box\Box P$ — modal logic S4
- **E** $\Diamond P \to \Box\Diamond P$ — modal logic S5

Semantics, more rigorously

The mathematical details

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where $\vDash$ respects connectives $\wedge, \vee, \neg, \rightarrow$, & satisfies:

Semantics, more rigorously

The mathematical details

Definition (Kripke frame)

A KRIPKE FRAME is a triple $(W, R, \vDash)$, where:

- $W :=$ set of "POSSIBLE WORLDS"
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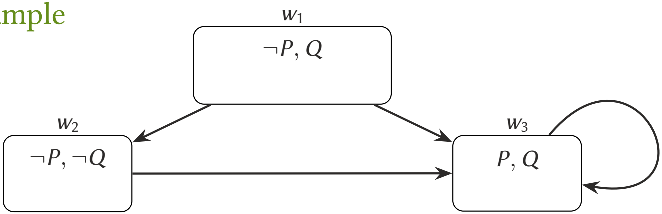
- $\models$ relates worlds to formulas — $w \models P$ means "P is true in w"

where $\models$ respects connectives $\wedge, \vee, \neg, \rightarrow$, & satisfies:

- $w \models \Box P$ if $w' \models P$ for EVERY w' such that wRw'
- $w \models \Diamond P$ if $w' \models P$ for SOME w' such that wRw'

Example



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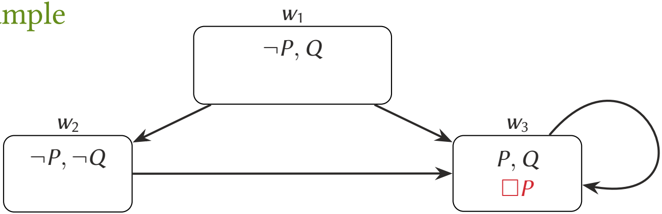

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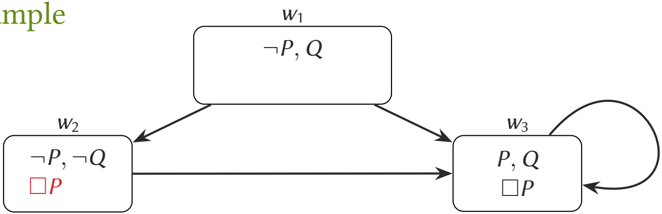

Semantics, more rigorously

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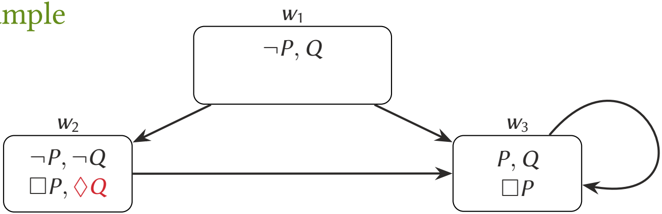
Example



Semantics, more rigorously

The mathematical details

Definition (Kripke frame)

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- $W :=$ set of "possible worlds"
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- $\vDash$ relates worlds to formulas — $w \vDash P$ means "P is true in w"

where $\vDash$ respects connectives $\wedge, \vee, \neg, \rightarrow$, & satisfies:

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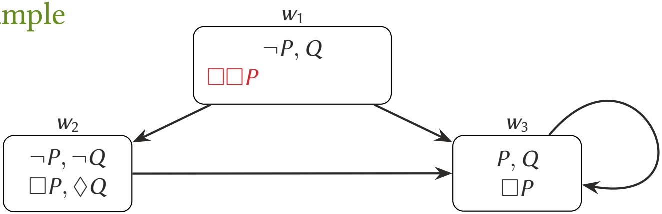
Example



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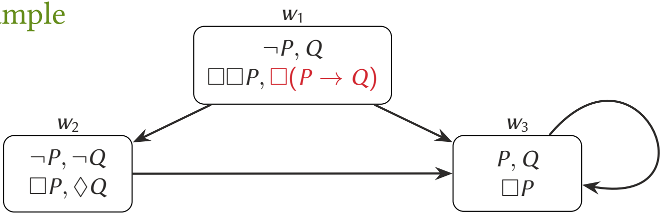
Example



Modal logic zoo

Logic	$\Box P$	$w_1 R w_2$	Restrictions on R	Axioms
alethic	P is necessary	w_2 is a (meta)physically possible alternative to w_1	reflexive, transitive, symmetric	D T 4 E

Modal logic zoo

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alethic	P is necessary	w_2 is a (meta)physically possible alternative to w_1	reflexive, transitive, symmetric	D T 4 E
deontic	P is morally obligatory	w_2 is morally preferable to w_1	total	D T 4 E

D: $\Box P \rightarrow \Diamond P$ is a weakened T: $\Box P \rightarrow P$

Applications of modal logic

*One system, **many** applications*

Modal logic zoo

Logic	$\Box P$	$w_1 R w_2$	Restrictions on R	Axioms
alethic	P is necessary	w_2 is a (meta)physically possible alternative to w_1	reflexive, transitive, symmetric	D T 4 E
deontic	P is morally obligatory	w_2 is morally preferable to w_1	total	D T 4 E
epistemic	P is known	w_2 is an alternative to w_1 consistent with knowledge	reflexive, transitive, symmetric	D T 4 E

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epistemic	P is known	w_2 is an alternative to w_1 consistent with knowledge	reflexive, transitive, symmetric	D T 4 E
doxastic	P is believed	w_2 is an alternative to w_1 consistent with belief	total, transitive, symmetric	D T 4 E

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Modal logic zoo

Logic	$\Box P$	$w_1 R w_2$	Restrictions on R	Axioms
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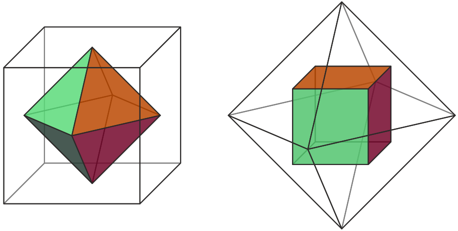
deontic	P is morally obligatory	w_2 is morally preferable to w_1	total	D T 4 E
epistemic	P is known	w_2 is an alternative to w_1 consistent with knowledge	reflexive, transitive, symmetric	D T 4 E
doxastic	P is believed	w_2 is an alternative to w_1 consistent with belief	total, transitive, symmetric	D T 4 E
temporal	P holds for all future time	w_2 is the world at a later time than w_1	dense, antisymmetric, total, transitive, …	*(various)*

D: $\Box P \to \Diamond P$ is a weakened T: $\Box P \to P$

4. Modal ontological argument



$\Diamond P$ $\Box P$

Theorem (Modal ontological argument)

In S5, we have

$$\Box(G \to \Box G),\ \Diamond G \quad \vdash \quad G.$$

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Let $G :=$ *"God exists" in the above.*

Theorem (Modal ontological argument)

In S5, we have

$$\Box(G \to \Box G),\ \Diamond G \quad \vdash \quad G.$$

Let $G :=$ "God exists" in the above.

$\longrightarrow$ *"If a NECESSARILY NECESSARY being is POSSIBLE, they EXIST"!*

A few lemmas

Lemma (Propositional logic results)

1. $P \to Q \qquad \vdash \; (\neg Q) \to (\neg P)$ *“contraposition”*

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1. $P \to Q \vdash (\neg Q) \to (\neg P)$ *"contraposition"*
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A few lemmas

Lemma (Propositional logic results)

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2. $P \vee Q,\ Q \to R \vdash P \vee R$ *"constructive dilemma"*
3. $P \vee Q,\ \neg Q \vdash P$ *"disjunction elimination"*

A few lemmas

Lemma (Propositional logic results)

1. $P \to Q \vdash (\neg Q) \to (\neg P)$ — *"contraposition"*
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3. $P \lor Q,\ \neg Q \vdash P$ — *"disjunction elimination"*
4. $\vdash P \lor \neg P$ — *"law of excluded middle"*

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Proof.

Invoke completeness of prop logic. ∎

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$\vdash \neg\Box Q \to \Box\neg\Box Q$

A few lemmas

Lemma (Propositional logic results)

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2. $P \vee Q,\ Q \to R \vdash P \vee R$ *"constructive dilemma"*
3. $P \vee Q,\ \neg Q \vdash P$ *"disjunction elimination"*
4. $\vdash P \vee \neg P$ *"law of excluded middle"*

Proof.

Invoke completeness of prop logic. ∎

Lemma (Contingency is necessary)

$\vdash \neg\Box Q \to \Box\neg\Box Q$

Proof.

1. $\Diamond(\neg Q) \to \Box\Diamond(\neg Q)$ by E $P = \neg Q$

A few lemmas

Lemma (Propositional logic results)

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2. $P \vee Q,\ Q \to R \vdash P \vee R$ *"constructive dilemma"*
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Invoke completeness of prop logic. ■

Lemma (Contingency is necessary)

$\vdash \neg\Box Q \to \Box\neg\Box Q$

Proof.

1. $\Diamond(\neg Q) \to \Box\Diamond(\neg Q)$ by E $P = \neg Q$

but $\Diamond\neg Q \equiv \neg\Box Q$, as $\Diamond = \neg\Box\neg$. ■

Proof of modal ontological argument

Theorem (Modal ontological argument)

In S5, we have

$$\Box(G \to \Box G),\ \Diamond G \quad \vdash \quad G.$$

Proof.

We first do some META-LOGICAL REASONING:

Proof of modal ontological argument

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1. $G \to \Box G \ \vdash\ \neg(\Box G) \to \neg G$ — by contraposition

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2. $\vdash (G \to \Box G) \to (\neg\Box G \to \neg G)$ — by deduction theorem

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3. $\vdash \Box((G \to \Box G) \to (\neg\Box G \to \neg G))$ — by rule of necessitation

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Now begin the FORMAL DEDUCTION:

1. $\Box(G \to \Box G)$ — assumption

Proof of modal ontological argument

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Proof.

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2. $\vdash (G \to \Box G) \to (\neg\Box G \to \neg G)$ — by deduction theorem
3. $\vdash \Box((G \to \Box G) \to (\neg\Box G \to \neg G))$ — by rule of necessitation
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2. $\Box(G \to \Box G) \to \Box(\neg\Box G \to \neg G)$ — proven above

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1. $\Box(G \to \Box G)$ — assumption
2. $\Box(G \to \Box G) \to \Box(\neg\Box G \to \neg G)$ — proven above
3. $\Box(\neg\Box G \to \neg G)$ — by *modus ponens* 1 & 2

Proof of modal ontological argument

Proof (*continued*).

1. $\Box(G \to \Box G)$ — assumption
2. $\Box(G \to \Box G) \to \Box(\neg\Box G \to \neg G)$ — proven above
3. $\Box(\neg\Box G \to \neg G)$ — by *modus ponens* 1 & 2

Proof of modal ontological argument

Proof (*continued*).

1. $\Box(G \to \Box G)$ — assumption
2. $\Box(G \to \Box G) \to \Box(\neg\Box G \to \neg G)$ — proven above
3. $\Box(\neg\Box G \to \neg G)$ — by *modus ponens* 1 & 2
4. $\Box(\neg\Box G \to \neg G) \to (\Box\neg\Box G \to \Box\neg G)$ — by K

Proof of modal ontological argument

Proof (*continued*).

1. $\Box(G \to \Box G)$ — assumption
2. $\Box(G \to \Box G) \to \Box(\neg\Box G \to \neg G)$ — proven above
3. $\Box(\neg\Box G \to \neg G)$ — by *modus ponens* 1 & 2
4. $\Box(\neg\Box G \to \neg G) \to (\Box\neg\Box G \to \Box\neg G)$ — by K
5. $\Box\neg\Box G \to \Box\neg G$ — by *modus ponens* 3 & 4

Proof of modal ontological argument

Proof (*continued*).

1. $\Box(G \to \Box G)$ — assumption
2. $\Box(G \to \Box G) \to \Box(\neg\Box G \to \neg G)$ — proven above
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4. $\Box(\neg\Box G \to \neg G) \to (\Box\neg\Box G \to \Box\neg G)$ — by K
5. $\Box\neg\Box G \to \Box\neg G$ — by *modus ponens* 3 & 4
6. $\Box G \vee \neg\Box G$ — by law of excluded middle

Proof of modal ontological argument

Proof (*continued*).

1. $\Box(G \to \Box G)$ — assumption
2. $\Box(G \to \Box G) \to \Box(\neg\Box G \to \neg G)$ — proven above
3. $\Box(\neg\Box G \to \neg G)$ — by *modus ponens* 1 & 2
4. $\Box(\neg\Box G \to \neg G) \to (\Box\neg\Box G \to \Box\neg G)$ — by K
5. $\Box\neg\Box G \to \Box\neg G$ — by *modus ponens* 3 & 4
6. $\Box G \vee \neg\Box G$ — by law of excluded middle
7. $\neg\Box G \to \Box\neg\Box G$ — proven in previous lemma

Proof of modal ontological argument

Proof (*continued*).

1. $\Box(G \to \Box G)$ — assumption
2. $\Box(G \to \Box G) \to \Box(\neg\Box G \to \neg G)$ — proven above
3. $\Box(\neg\Box G \to \neg G)$ — by *modus ponens* 1 & 2
4. $\Box(\neg\Box G \to \neg G) \to (\Box\neg\Box G \to \Box\neg G)$ — by K
5. $\Box\neg\Box G \to \Box\neg G$ — by *modus ponens* 3 & 4
6. $\Box G \vee \neg\Box G$ — by law of excluded middle
7. $\neg\Box G \to \Box\neg\Box G$ — proven in previous lemma
8. $\Box G \vee \Box\neg\Box G$ — by constructive dilemma 6 & 7

Proof of modal ontological argument

Proof (*continued*).

1	$\Box(G \to \Box G)$	assumption
2	$\Box(G \to \Box G) \to \Box(\neg\Box G \to \neg G)$	proven above
3	$\Box(\neg\Box G \to \neg G)$	by *modus ponens* 1 & 2
4	$\Box(\neg\Box G \to \neg G) \to (\Box\neg\Box G \to \Box\neg G)$	by K
5	$\Box\neg\Box G \to \Box\neg G$	by *modus ponens* 3 & 4
6	$\Box G \vee \neg\Box G$	by law of excluded middle
7	$\neg\Box G \to \Box\neg\Box G$	proven in previous lemma
8	$\Box G \vee \Box\neg\Box G$	by constructive dilemma 6 & 7
9	$\Box G \vee \Box\neg G$	by constructive dilemma 5 & 8

Proof of modal ontological argument

Proof (*continued*).

1	$\Box(G \to \Box G)$	assumption
2	$\Box(G \to \Box G) \to \Box(\neg\Box G \to \neg G)$	proven above
3	$\Box(\neg\Box G \to \neg G)$	by *modus ponens* 1 & 2
4	$\Box(\neg\Box G \to \neg G) \to (\Box\neg\Box G \to \Box\neg G)$	by K
5	$\Box\neg\Box G \to \Box\neg G$	by *modus ponens* 3 & 4
6	$\Box G \lor \neg\Box G$	by law of excluded middle
7	$\neg\Box G \to \Box\neg\Box G$	proven in previous lemma
8	$\Box G \lor \Box\neg\Box G$	by constructive dilemma 6 & 7
9	$\Box G \lor \Box\neg G$	by constructive dilemma 5 & 8
10	$\neg\Box\neg G$	equivalent to assumption $\Diamond G$

Proof of modal ontological argument

Proof (*continued*).

1	$\Box(G \to \Box G)$	assumption
2	$\Box(G \to \Box G) \to \Box(\neg\Box G \to \neg G)$	proven above
3	$\Box(\neg\Box G \to \neg G)$	by *modus ponens* 1 & 2
4	$\Box(\neg\Box G \to \neg G) \to (\Box\neg\Box G \to \Box\neg G)$	by K
5	$\Box\neg\Box G \to \Box\neg G$	by *modus ponens* 3 & 4
6	$\Box G \lor \neg\Box G$	by law of excluded middle
7	$\neg\Box G \to \Box\neg\Box G$	proven in previous lemma
8	$\Box G \lor \Box\neg\Box G$	by constructive dilemma 6 & 7
9	$\Box G \lor \Box\neg G$	by constructive dilemma 5 & 8
10	$\neg\Box\neg G$	equivalent to assumption $\Diamond G$
11	$\Box G$	by disjunction elimination 9 & 10

Proof of modal ontological argument

Proof (*continued*).

1	$\Box(G \to \Box G)$	assumption
2	$\Box(G \to \Box G) \to \Box(\neg\Box G \to \neg G)$	proven above
3	$\Box(\neg\Box G \to \neg G)$	by *modus ponens* 1 & 2
4	$\Box(\neg\Box G \to \neg G) \to (\Box\neg\Box G \to \Box\neg G)$	by K
5	$\Box\neg\Box G \to \Box\neg G$	by *modus ponens* 3 & 4
6	$\Box G \vee \neg\Box G$	by law of excluded middle
7	$\neg\Box G \to \Box\neg\Box G$	proven in previous lemma
8	$\Box G \vee \Box\neg\Box G$	by constructive dilemma 6 & 7
9	$\Box G \vee \Box\neg G$	by constructive dilemma 5 & 8
10	$\neg\Box\neg G$	equivalent to assumption $\Diamond G$
11	$\Box G$	by disjunction elimination 9 & 10
12	$\Box G \to G$	by T

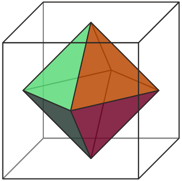
Proof of modal ontological argument

Proof (*continued*).

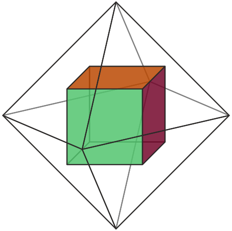
1	$\Box(G \to \Box G)$	assumption
2	$\Box(G \to \Box G) \to \Box(\neg\Box G \to \neg G)$	proven above
3	$\Box(\neg\Box G \to \neg G)$	by *modus ponens* 1 & 2
4	$\Box(\neg\Box G \to \neg G) \to (\Box\neg\Box G \to \Box\neg G)$	by K
5	$\Box\neg\Box G \to \Box\neg G$	by *modus ponens* 3 & 4
6	$\Box G \vee \neg\Box G$	by law of excluded middle
7	$\neg\Box G \to \Box\neg\Box G$	proven in previous lemma
8	$\Box G \vee \Box\neg\Box G$	by constructive dilemma 6 & 7
9	$\Box G \vee \Box\neg G$	by constructive dilemma 5 & 8
10	$\neg\Box\neg G$	equivalent to assumption $\Diamond G$
11	$\Box G$	by disjunction elimination 9 & 10
12	$\Box G \to G$	by T
13	G	by *modus ponens* 11 & 12

∎

5. Conclusion



$\Diamond P$



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- We may entertain possible worlds where LAWS OF LOGIC fail

  $\longrightarrow$ "NON-NORMAL modal logics", lacking law of necessitation

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- Yet there are replies to these parodies; see Stacey (2023)

In short, the jury is out, & the literature vast.

References I

- For *modal logic* in general, see ch. 2 & 6 of **De Swart (2018)**, especially ex. 6.3–6.5.
- For *modal ontological argument*, see papers by **Kane (1984)** & **Stacey (2023)**

Full reference list

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