Dario Trinchero

Work supervised by F. G. Scholtz

Room 316 Talk:

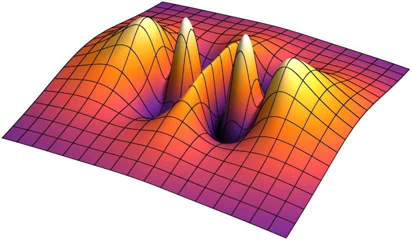
Pinhole Interference in 3D Fuzzy Space

A natural quantum-to-classical transition

Stellenbosch University
August 2023

Outline

The Setup



Motivation & Goal

A lofty question

Question:

Can the structure of spacetime at SMALLEST LENGTH SCALE affect the physics we perceive at LARGER LENGTH SCALES (*i.e. classical physics*)?

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A concrete investigation

Our investigation:

Study DOUBLE-PINHOLE SETUP in the 3D FUZZY SPACE formlism of NON-COMMUTATIVE quantum mechanics.

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- Otherwise ORDINARY QUANTUM MECHANICS !

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Why this particular investigation?

Why non-commutative geometry?

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Why pinhole interference?

- Illustrative TOY MODEL ⟵ quantum behaviour = interference
- QUANTIFIABLE SUPPRESSION strength
- Good setup for EXPERIMENTAL TESTING

Pittaway & Scholtz (2021)

2-slits in non-commutative 2D Moyal plane

Quantum-to-Classical Transition

Existing work in lower dimensions

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- Moyal plane interference pattern:
  - is ASYMMETRIC under reflection ⟵ ∵ Moyal commutation relations break rotational symmetry
  - has UNOBSERVABLE SUPPRESSION

Moyal plane interference

$$P(\boldsymbol{D}) = 1 + \underbrace{\exp\left[-\frac{N\theta m^2}{2\hbar^2}\boldsymbol{v}^2(1-\cos\alpha)\right]}_{\text{interference suppression}} \underbrace{\cos\left(\cdots\right)}_{\text{interference}}$$

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Quantum-to-Classical Transition

Why Moyal plane suppression is unobservable

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- $m \sim 10^{-31}$ kg $\longleftarrow$ mass of proton
- $\sqrt{\theta} \sim 10^{-35}$ m $\longleftarrow$ Planck length
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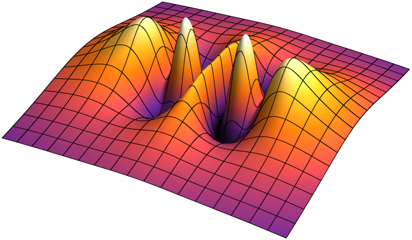
$\Longrightarrow \|\boldsymbol{v}\| \gtrsim 10^{16}\,\mathrm{m\,s^{-1}}$!

The Formalism



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Definition (Fuzzy space)

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$\Longrightarrow$ 1 copy of each quantised radius

Definition (Quantum state space)

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$$\longrightarrow \quad \mathcal{H}_\mathrm{q} := \left\{ \begin{matrix} \text{operators on } \mathcal{H}_\mathrm{c} \\ \text{generated by } \hat{x}_i \end{matrix} \right\}$$

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so $\left\| \hat{\mathrm{Proj}}_{[1]\oplus\cdots\oplus[N]} \right\| \sim \frac{4}{3}\pi \left[\lambda(N+1)\right]^3$

Observables

Just like normal QM

Definition (Observables)

Hermitian operators on $\mathcal{H}_q$

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HERMITIAN OPERATORS on $\mathcal{H}_q$

Important observables

- POSITION $\longrightarrow$ $\hat{X}_i \, |\psi) := |\hat{x}_i \psi)\,,\ \hat{R}\,|\psi) := |\hat{r}\psi)$ $\longleftarrow$ like normal QM

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- HAMILTONIAN $\longrightarrow$ $\hat{H} = -\frac{\hbar^2}{2m}\hat{\Delta} + V(\hat{R})$ $\longleftarrow$ like normal QM

Physical Subspace

Physical subspace

$$\mathcal{H}_q = \bigoplus_{n\in\mathbb{N}} [\boldsymbol{n}] \otimes [\boldsymbol{n}]^* \subset \mathcal{H}_c \otimes \mathcal{H}_c^*$$

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Projection onto $\mathcal{H}_\mathrm{q}$

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2. KERNEL $\longrightarrow$ $\mathcal{H}_\mathrm{q} = \ker \hat{\Gamma}$
3. PROJECTION $\longrightarrow$ $\hat{Q} := \frac{1}{2\pi}\int_0^{2\pi} e^{i\phi\hat{\Gamma}}\, d\phi$

Position Measurement

… as a POVM

Definition (Position measurement)

1. Position EIGENSTATES $\longrightarrow$ MINIMUM-UNCERTAINTY states

$$|\boldsymbol{z}\rangle := e^{-\frac{1}{2}\bar{z}_\alpha z_\alpha} e^{z_\alpha a^\dagger_\alpha} |0\rangle \,, \text{ for}$$

$$\boldsymbol{z} := e^{i\gamma}\sqrt{\frac{r}{\lambda}} \begin{bmatrix} \cos\left(\frac{\theta}{2}\right) e^{-i\frac{\phi}{2}} \\ \sin\left(\frac{\theta}{2}\right) e^{i\frac{\phi}{2}} \end{bmatrix} \in \mathbb{C}^2 \quad \longleftarrow \quad \text{encodes } \boldsymbol{D} = (r, \theta, \phi)$$

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2 POVM

$$\longrightarrow \qquad |z_1, z_2, n_1, n_2)_{\text{ph}} := \hat{Q} \frac{1}{\sqrt{4\pi\lambda^2\hat{r}}} |\boldsymbol{z}\rangle\langle n_1, n_2|$$

$$\longrightarrow \qquad \hat{\Pi}_{\boldsymbol{z}} := \sum_{n_1, n_2} |z_1, z_2, n_1, n_2)_{\text{ph ph}}(z_1, z_2, n_1, n_2|,$$

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3. BORN RULE $\longrightarrow$ $P(\boldsymbol{D}) = \mathrm{Tr}_{\mathrm{q}}\left(\hat{\Pi}_{\boldsymbol{z}}\rho\right)$

Position Measurement

… as weak measurement

Weak measurement picture

1. $\mathcal{H}_\mathrm{q} \subset \mathcal{H}_\mathrm{c} \otimes \mathcal{H}_\mathrm{c}^*$

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Weak measurement picture

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2. $\hat{X}_i$ act only on $\mathcal{H}_\mathrm{c}$
3. $\therefore$ POSITION measurement

$\longrightarrow$ LOCAL measurement

$\longrightarrow$ traces out "ENVIRONMENT", $\mathcal{H}_\mathrm{c}^*$

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Coordinate Representation

The analogue of wavefunctions

Definition (Symbol & star product)

1. Position-encoding states $\longrightarrow$ $|\boldsymbol{z}) := \hat{Q}\dfrac{1}{\sqrt{4\pi\lambda^2\hat{r}}}\,|\boldsymbol{z}\rangle\langle\boldsymbol{z}|$

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2. Coordinate rep $\longrightarrow$ $\psi(\boldsymbol{z}) := (\boldsymbol{z}|\psi) = \langle\boldsymbol{z}|\sqrt{4\pi\lambda^2 \hat{r}}\,\psi|\boldsymbol{z}\rangle$

Coordinate Representation

The analogue of wavefunctions

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ψ is indep of γ

$\therefore$ function on $\mathbb{R}^3$

Coordinate Representation

The analogue of wavefunctions

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3. Symbol $\longrightarrow$ $\langle\boldsymbol{z}|\psi|\boldsymbol{z}\rangle$

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Coordinate Representation

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3. Symbol $\longrightarrow$ $\langle\boldsymbol{z}|\psi|\boldsymbol{z}\rangle$
4. Voros product $\longrightarrow$ $\star := \exp\left[\overleftarrow{\partial}_{z_\alpha}\overrightarrow{\partial}_{\bar{z}_\alpha}\right]$

Notable properties

1. COMPLETENESS $\longrightarrow$ $\displaystyle\int \frac{d^4z}{\pi^2}\,|\boldsymbol{z})\,\bar{\star}\,(\boldsymbol{z}| = \mathbf{1}_{\mathrm{q}}$

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Wave-Mechanics

An alternative development of our formalism

Notable properties

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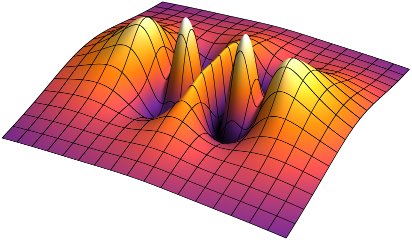
4. EQUIVALENCE $\longrightarrow$ $\langle \boldsymbol{z}|\psi|\boldsymbol{z}\rangle$ fully determines $|\psi)$

$\Longrightarrow$ Alternate "wave-mechanics" development !

Free Particle Solutions



Non-Commutative Free Schrödinger Equation

Non-commutative free Schrödinger equation

$$\hat{H}\,|\psi) = -\frac{\hbar^2}{2m}\hat{\Delta}\,|\psi) = E\,|\psi)$$

Non-Commutative Free Schrödinger Equation

Non-commutative free Schrödinger equation

$$\hat{H}\,|\psi) = -\frac{\hbar^2}{2m}\hat{\Delta}\,|\psi) = E\,|\psi)$$

Types of solutions:

1. PLANE WAVE $\longrightarrow$ $|\boldsymbol{k}) := e^{i\boldsymbol{k}\cdot\hat{\boldsymbol{x}}}$ $\longleftarrow$ typical form
2. SPHERICAL WAVE $\longrightarrow$ $|k, l, m) = g_l(\hat{n} - l, k)\,\hat{\mathcal{Y}}_{lm}$

Plane wave solutions

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Properties

- Represent (all) SU(2) ELEMENTS
- COMPOSE "nicely" $\longrightarrow$ $e^{i\boldsymbol{k}_1\cdot\hat{\boldsymbol{x}}}e^{i\boldsymbol{k}_2\cdot\hat{\boldsymbol{x}}} = e^{i\boldsymbol{k}_3\cdot\hat{\boldsymbol{x}}}$

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$$\cos(\lambda k_3) = \cos(\lambda k_1)\ \cos(\lambda k_2) - \hat{\boldsymbol{k}}_1 \cdot \hat{\boldsymbol{k}}_2 \sin(\lambda k_1)\sin(\lambda k_2),$$

$$\sin(\lambda k_3)\hat{\boldsymbol{k}}_3 = \hat{\boldsymbol{k}}_1 \sin(\lambda k_1)\cos(\lambda k_2) + \hat{\boldsymbol{k}}_2 \sin(\lambda k_2)\cos(\lambda k_1) - \hat{\boldsymbol{k}}_1 \times \hat{\boldsymbol{k}}_2 \sin(\lambda k_1)\sin(\lambda k_2)$$

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- DISPERSION RELATION $\longrightarrow$ $\hat{H}\,|\boldsymbol{k}) = \dfrac{2\hbar^2}{m\lambda^2}\sin^2\left(\dfrac{k\lambda}{2}\right)|\boldsymbol{k})$

Spherical wave solutions

2 SPHERICAL WAVE $\longrightarrow$ $|k, l, m) = g_l(\hat{n} - l, k)\, \hat{\mathcal{Y}}_{lm}$

Spherical wave solutions

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Properties

- Permit 2 RADIAL SOLUTIONS $\longrightarrow$ $g_l \quad = \; A\, g_{J,l} + \mathrm{B}\, g_{Y,l}$

  c.f. spherical Bessel- & Neumann

Spherical wave solutions

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- ASYMPTOTICALLY $\longrightarrow$ $$g_{H,l}(n,k) \sim \frac{e^{i\lambda(n+l+1)k}}{(in)^{l+1}}$$

$$\therefore\ \langle \boldsymbol{z}|g_{H,l}(\hat{n},k)|\boldsymbol{z}\rangle \sim \frac{e^{r(\cos(\lambda k)+i\sin(\lambda k)-1)/\lambda}}{(ir/\lambda)^{l+1}}$$

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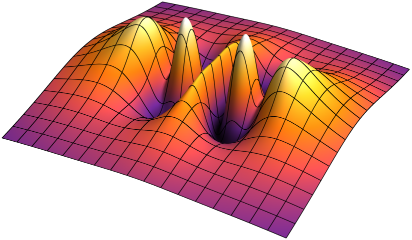
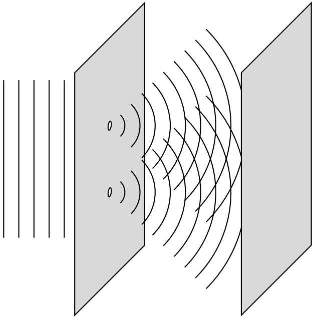
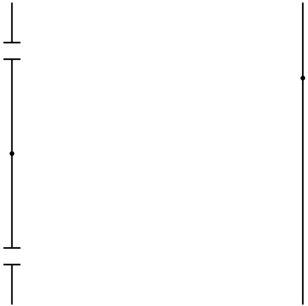
Main Calculation



Double-Pinhole Setup

The most famous quantum experiment





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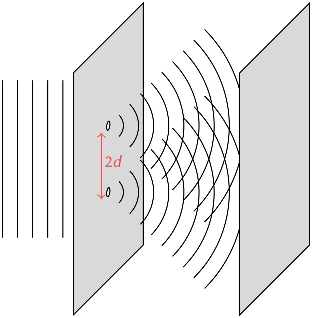

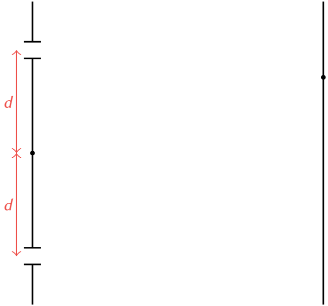

- Pinholes $\longrightarrow$ $z = \pm d$

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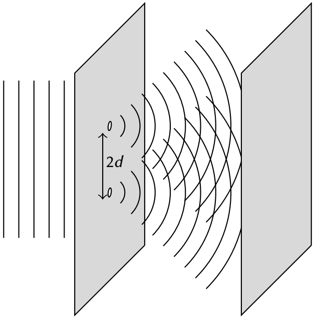

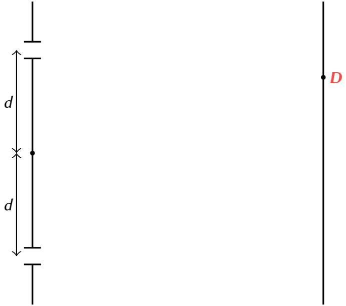

- PINHOLES $\longrightarrow$ $z = \pm d$
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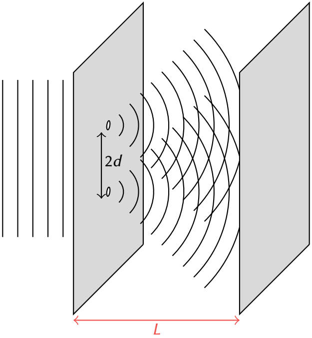

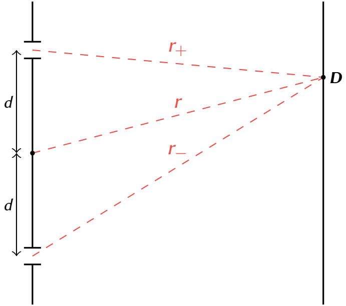

- PINHOLES $\longrightarrow$ $z = \pm d$
- DETECTION POINT $\longrightarrow$ $\boldsymbol{D} = (L, y_D, z_D) \equiv (r, \theta, \phi)$
- DISTANCES $\longrightarrow$ $L, r, r_\pm$ $\longleftarrow$ each $\gg d$: large separation approx

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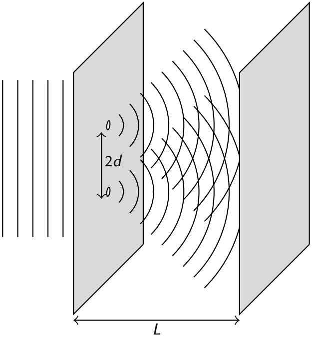
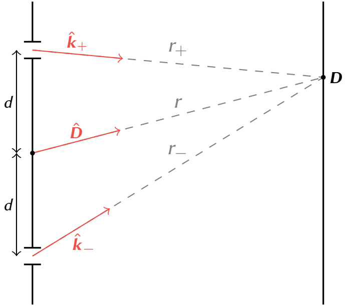
The most famous quantum experiment





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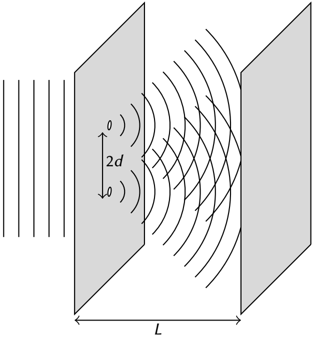
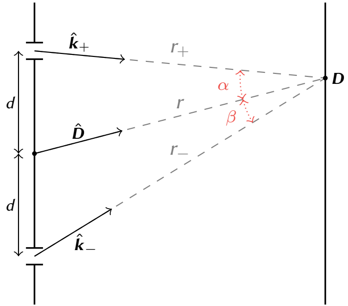
- Distances $\longrightarrow$ $L, r, r_\pm$
- Wavenumbers $\longrightarrow$ $\boldsymbol{k}_\pm = k\,\hat{\boldsymbol{k}}_\pm$

Double-Pinhole Setup

The most famous quantum experiment





- Pinholes $\longrightarrow$ $z = \pm d$
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- Wavenumbers $\longrightarrow$ $\boldsymbol{k}_\pm = k\,\hat{\boldsymbol{k}}_\pm$
- Angles $\longrightarrow$ α, β

Interference calculation overview

1. STATE at ***D*** $\longrightarrow \quad \psi \sim \begin{Bmatrix} \text{spherical wave} \\ \text{from } z = +d \end{Bmatrix} + \begin{Bmatrix} \text{spherical wave} \\ \text{from } z = -d \end{Bmatrix}$

Interference Calculation

Broad strategy

Interference calculation overview

1. STATE at $\boldsymbol{D}$ $\longrightarrow$ $\psi \sim \begin{Bmatrix} \text{spherical wave} \\ \text{from } z = +d \end{Bmatrix} + \begin{Bmatrix} \text{spherical wave} \\ \text{from } z = -d \end{Bmatrix}$

2. PARAXIAL approximation $\longrightarrow$ $\begin{Bmatrix} \text{spherical wave} \\ \text{from } z = \pm d \end{Bmatrix} \sim \dfrac{1}{r_\pm} e^{i\boldsymbol{k}_\pm \cdot \boldsymbol{D}}$

Interference Calculation

Broad strategy

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3. BORN RULE $\longrightarrow$ $P(\boldsymbol{D}) = \mathrm{Tr}\left(\hat{\Pi}_{\boldsymbol{D}} \rho\right)$

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Interference Calculation

Overview of commutative treatment

Commutative interference calculation

1. State at $\boldsymbol{D}$ $\longrightarrow$ $\psi(\boldsymbol{D}) \sim \frac{1}{r_+} e^{ikr_+} + \frac{1}{r_-} e^{ikr_-}$ $\longleftarrow$ asymptotic form of spherical Hankel

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2. Paraxial approximation $\longrightarrow$ $\frac{1}{r_\pm} e^{ikr_\pm} \sim \frac{1}{r_\pm} e^{i\boldsymbol{k}_\pm \cdot \boldsymbol{D}}$

Commutative interference calculation

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Interference Calculation

Overview of commutative treatment

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$$P_{\text{comm}}(\boldsymbol{D}) \sim \frac{1}{r_+ r_-} \Bigg[\underbrace{\frac{2d^2}{r_+ r_-} + \cos(\alpha + \beta)}_{\text{bimodal shaping function}} + \underbrace{\cos(rk(\cos\alpha - \cos\beta))}_{\text{interference terms}} \Bigg]$$

Interference Calculation

Overview of non-commutative treatment

Non-commutative interference calculation

1. Symbol at D $\longrightarrow$ $\langle z|\psi|z\rangle \sim \left\langle z^{+}\middle|g_k(\hat{n})\middle|z^{+}\right\rangle + \left\langle z^{-}\middle|g_k(\hat{n})\middle|z^{-}\right\rangle$

Interference Calculation

Overview of non-commutative treatment

Non-commutative interference calculation

1. SYMBOL at $\boldsymbol{D}$ $\longrightarrow$ $\langle \boldsymbol{z}|\psi|\boldsymbol{z}\rangle \sim \left\langle \boldsymbol{z}^{+}\middle|g_k(\hat{n})\middle|\boldsymbol{z}^{+}\right\rangle + \left\langle \boldsymbol{z}^{-}\middle|g_k(\hat{n})\middle|\boldsymbol{z}^{-}\right\rangle$
2. PARAXIAL approximation $\longrightarrow$ $\left\langle \boldsymbol{z}^{\pm}\middle|g_k(\hat{n})\middle|\boldsymbol{z}^{\pm}\right\rangle \sim \langle \boldsymbol{z}|\eta_{\pm}\, e^{i\boldsymbol{k}_{\pm}\cdot\hat{\boldsymbol{x}}}|\boldsymbol{z}\rangle$

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Interference Calculation

Overview of non-commutative treatment

Non-commutative interference calculation

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3. STATE $\longrightarrow$ $|\psi) \sim \eta_+\, e^{i\boldsymbol{k}_+\cdot\hat{\boldsymbol{x}}} + \eta_-\, e^{i\boldsymbol{k}_-\cdot\hat{\boldsymbol{x}}}$

Non-commutative interference calculation

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Non-commutative interference calculation

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3. STATE $\longrightarrow$ $|\psi) \sim \eta_+\, e^{i\boldsymbol{k}_+\cdot\hat{\boldsymbol{x}}} + \eta_-\, e^{i\boldsymbol{k}_-\cdot\hat{\boldsymbol{x}}}$
4. BORN RULE $\longrightarrow$ $P(\boldsymbol{D}) = \mathrm{Tr}_q\left(\hat{\Pi}_{\boldsymbol{z}}\ |\psi)(\psi|\right)$
5. Compute remaining MATRIX ELEMENTS

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Interference Calculation

The main result!

$$P(\boldsymbol{D}) \sim \underbrace{\frac{\eta_+^2 + \eta_-^2}{2}\left(\frac{r}{\lambda} + 1\right)}_{\text{bimodal shaping function}} + \underbrace{\eta_+ \eta_- e^{A - r/\lambda}}_{\text{interference suppression}} \times \underbrace{((A+1)\cos B - B \sin B)}_{\text{interference terms}}$$

$$\text{where} \begin{cases} \eta_\pm & := \frac{\lambda}{r_\pm} \exp\left[\frac{1}{\lambda}(r_\pm - r)(\cos(\lambda k) - 1)\right], \\ A & := \frac{r}{\lambda}\left(\cos^2(\lambda k) + \cos(\alpha + \beta)\sin^2(\lambda k)\right), \\ B & := \frac{r}{\lambda} \sin(\lambda k)\cos(\lambda k)(\cos\alpha - \cos\beta) \end{cases}$$

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Interference Calculation

The main result!

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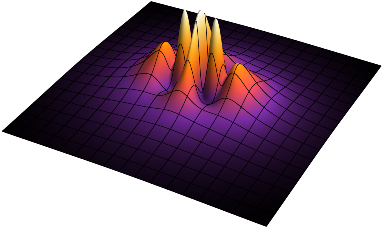
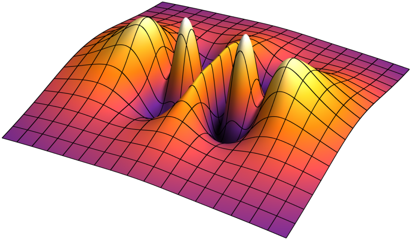
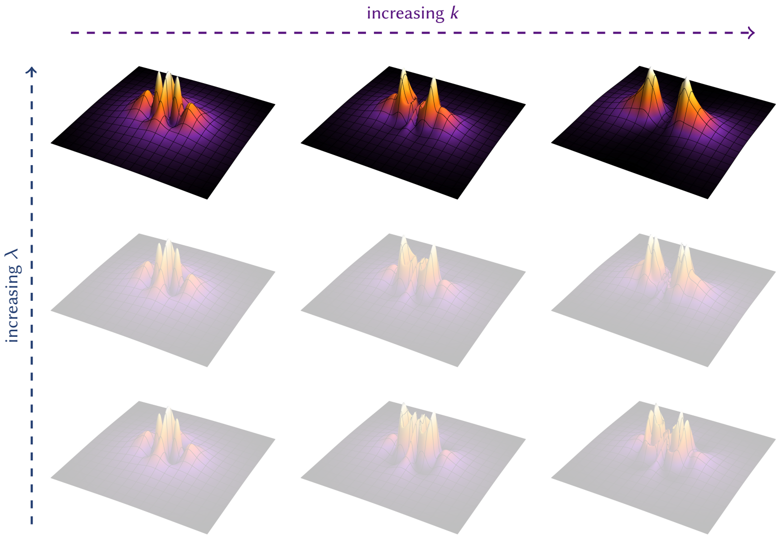


Discussion of Results



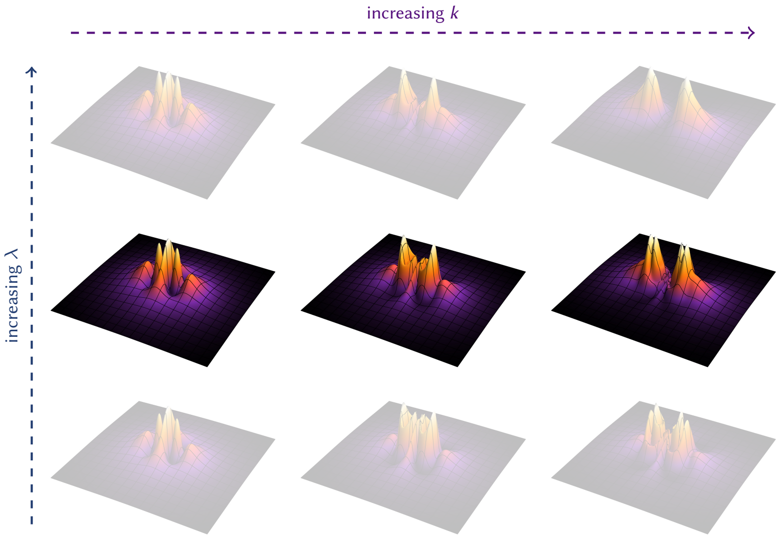
Qualitative Behaviour

& commutative limit



Qualitative Behaviour

& commutative limit



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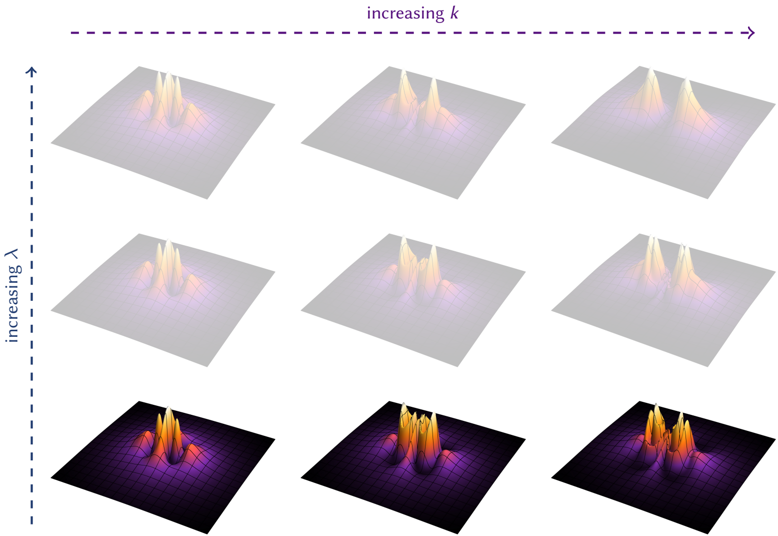

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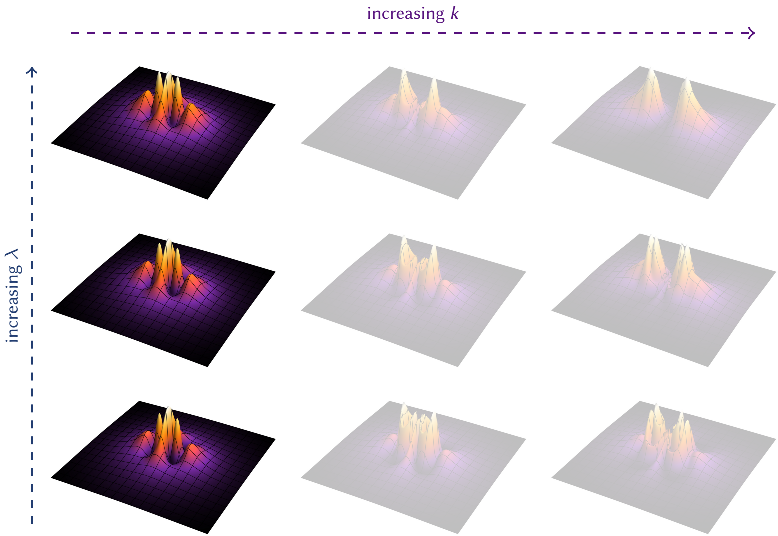

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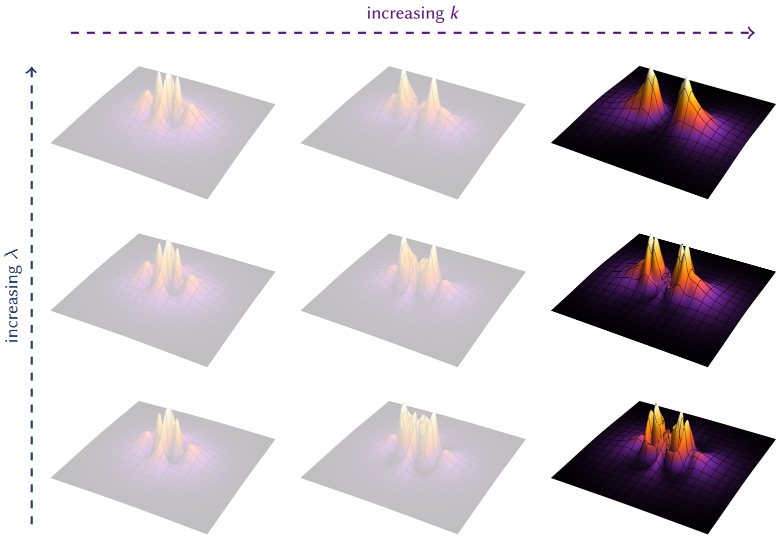

Fuzzy space interference

$$P(\boldsymbol{D}) \sim \cdots + \underbrace{\eta_+\eta_- \exp\left[\frac{r}{\lambda}\sin^2(\lambda k)(\cos(\alpha+\beta)-1)\right]}_{\text{interference suppression}} \underbrace{(\cdots)}_{\text{interference}}$$

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Important features

- r dependence
- suppression possible at low k

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Extending definitions

N particles

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4. Plane waves $\longrightarrow$ $\left|\boldsymbol{k}^{(i\cdots N)}\right) = \exp\left[i\sum_{n=1}^{N}\boldsymbol{k}^{(n)}\cdot\hat{\boldsymbol{x}}^{(n)}\right]$

$\vdots$

Macroscopic Scaling

Center-of-mass coordinates

N particles

5 Center-of-mass frame

$$\longrightarrow \quad \hat{\boldsymbol{x}}^{(\mathrm{CM})} := \frac{1}{N}\sum_{n=1}^{N}\hat{\boldsymbol{x}}^{(n)}, \qquad \hat{\boldsymbol{\xi}}^{(n)} := \hat{\boldsymbol{x}}^{(n)} - \hat{\boldsymbol{x}}^{(\mathrm{CM})}$$

$$\boldsymbol{k}^{\mathrm{tot}} := \sum_{n=1}^{N}\boldsymbol{k}^{(n)}, \qquad \boldsymbol{q}^{(n)} := \boldsymbol{k}^{(n)} - \frac{1}{N}\boldsymbol{k}^{\mathrm{tot}}$$

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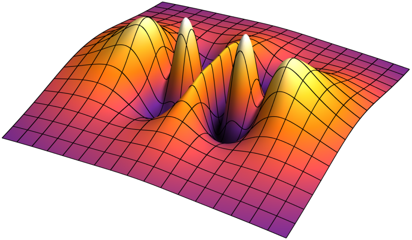
Concluding Remarks



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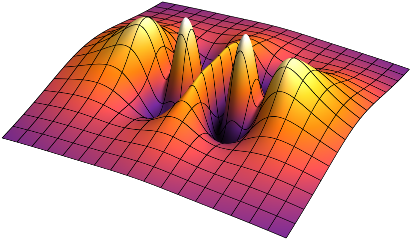
   - *Kleckner et al. (2008)* ⟶ describe entangling photon with macroscopic cantilever

Summary

Key takeaways

- Fuzzy space $\longrightarrow$ CLASSICAL TRANSITION without heat bath
- Quantum suppression $\longrightarrow$ realistically OBSERVABLE
- Suppression strength $\longrightarrow$ EXTENSIVE & DISTANCE-dependent



Future Work Proposals

1. ALTERNATE SETUPS:
   - Treat fuzzy-space VON NEUMANN MEASUREMENT
2. FORMALISM EXTENSIONS:
   - Extend to non-commutative QFT
3. EXPERIMENTAL VERIFICATION:
   - Implement proposed experiment
   - Devise alternate experiment

Accompanying Article

Trinchero, D., & Scholtz, F. G. (2023, March).
Pinhole interference in three-dimensional fuzzy space.
Annals of Physics, 450, 169224.
(arXiv:2212.01449 [quant-ph])

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